THE SUPERSYMMETRIC CP PROBLEM IN ORBIFOLD COMPACTIFICATIONS

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ABSTRACT

The possibility of spontaneous breaking of CP symmetry by the expectation values of orbifold moduli is investigated with particular reference to CP violating phases in soft supersymmetry breaking terms. The effect of different mechanisms for stabilizing the dilaton and the form of the non-perturbative superpotential on the existence and size of these phases is studied. Models with modular symmetries which are subgroups of $PSL(2, Z)$, as well as the single overall modulus T case with the full $PSL(2, Z)$ modular symmetry, are discussed. Non-perturbative superpotentials involving the absolute modular invariant $j(T)$, such as may arise from F-theory compactifications, are considered.

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1 Introduction

The concept of *symmetry* has been the most useful guiding principle in our search for discovering the fundamental laws of Nature. However, the understanding of *symmetry breaking* stands on an equal footing since a lot of complex phenomena in our cosmos depend on it. The observed baryon asymmetry of the Universe, if dynamically generated, requires that CP is violated. The observed tiny breaking of CP [1] symmetry is one of the most intriguing problems in particle physics and it has now been with us for more than 30 years, awaiting an explanation. It therefore constitutes one of the most promising directions in the search for new physics beyond the Standard Model.

String theory may provide a new perspective on the longstanding question of the origin of CP violation. It has been argued [2] that there is no explicit CP symmetry breaking in string theory whether perturbative or non-perturbative. However, CP violation might arise from complex expectation values of moduli or other scalars [2]-[5]. In all supergravity theories, including those derived from string theory, there is the possibility of CP violating phases in the soft supersymmetry breaking A and B terms and gaugino masses which are in addition to a possible phase in the Kobayashi-Maskawa matrix and the θ parameter of QCD; (see for example, [6] and references therein.) In compactifications of string theory, soft supersymmetry breaking terms can be functions of moduli such as those associated with the radius and angles characterizing the underlying torus of the orbifold compactification. Then, if these moduli develop complex vacuum expectation values this can feed through to the low energy supergravity as CP violating phases.

Indeed, any nonzero value for d_n, the neutron electric dipole moment, is an indication of CP violation. In principle, complex soft superymmetry breaking terms in supersymmetric (SUSY) theories and their resulting phases can lead to large contributions to d_n [7]. These phases are constrained by experiment to be $\leq O(10^{-3})$. It is therefore a serious challenge for SUSY theories to explain why these phases are so small, i.e why soft susy breaking terms preserve CP to such a high degree [8]. As we will show below, string supersymmetric theories relate the required smallness of CP phases to properties of modular functions.

To estimate the size of such CP violating phases for orbifold compactifications, it is first necessary to minimize the effective potential to determine the expectation values of the moduli fields T_i. Such calculations may be sensitive to the solution proposed to the problem of stabilizing the dilaton expectation value. Two mechanisms for stabilizing the dilaton have been proposed. The first one assumes that more than one gaugino condensate [9] is present in the superpotential W_{np}. This class of models has been termed as multiple gaugino condensate or racetrack models. The second proposal is more *stringy* in nature [10, 11]. There are good reasons to believe that stringy non-perturbative corrections to the non-holomorphic Kähler potential are sizeable and can stabilize the dilaton, thereby solving the runaway problem.

Modular invariance of the effective Lagrangian strongly restricts the form of the non-perturbative superpotential W_{np} and connects it to the theory of modular forms [12]. Recently, it has been demonstrated that superpotentials with modular properties may also arise in F-theory compactifications [13]. As has been shown in [14], the superpotential W is a section of a holomorphic line bundle. Among other ways, theta functions can be viewed as sections of line bundles on abelian varieties and/or the moduli space of abelian varieties [15]. In F-theory constructions [13] non-trivial non-perturbative superpotentials involve theta functions, in fact an E_8 theta function in

the example of [13], and as such they exhibit definite modular properties. These recent developements enhance the expectations that the superpotential is indeed a *modular form* under duality transformations. In this work we investigate the effect that modular invariance and dilaton-stabilization by the mechanisms described above have on the CP properties of the resulting soft supersymmetry-breaking terms.

The material of this paper is organized as follows. In section 2 we describe the form of the effective potential that encompasses both possibilities for the stabilization of the dilaton, and present the soft supersymmetry-breaking terms emerging from such a potential. In section 3 we study the case of the $Z6 - IIb$ orbifold with modular symmetries for some of its moduli which are subgroups of $PSL(2, Z)$, and assuming that the dilaton is stabilised by a multiple gaugino condensate. In section 4 we study the case of the $Z6 - IIb$ orbifold where non-perturbative stringy corrections to the Kähler potential are responsible for the stabilization of the dilaton and present our results. In section 5 the case of the overall modulus T with a full $PSL(2, Z)$ modular symmetry is presented. Our conclusions are presented in section 6.

2   Effective potential and soft supersymmetry breaking terms

To estimate the size of CP violating phases in the soft supersymmetry-breaking terms, we need the form of the effective potential V_{eff}. We can then minimize to find the expectation values of the T moduli, which are in general complex. The outcome may be sensitive to the solution proposed to the problem of stabilizing the dilaton expectation value at a realistic value of ReS. The possibilities we shall consider are that this stabilization is due to a multiple gaugino condensate in the hidden sector [9] (including hidden sector matter), or due to stringy non-perturbative corrections to the Kähler potential [10, 11]. For convenience, we write down expressions for the effective potential and for the soft supersymmetry breaking terms that are general enough to encompass both possibilities.

The general form of the multiple gaugino condensate non-perturbative superpotential W_{np} derived from orbifold compactifications is [12, 16]

$$
\begin{aligned}
W_{np} &= \sum_a h_a\, e^{\frac{24\pi^2 S}{b_a}} \prod_{i,m} \Big(\eta\Big(\frac{T_i}{l_{im}}\Big)\Big)^{-c_{im}(1-\frac{3\delta_{GS}^i}{b_a})} \\
&\times \prod_j H_j(T_j)
\end{aligned}
\tag{1}
$$

for some coefficients h_a, where δ_{GS}^i are Green-Schwarz parameters and b_a are renormalization group coefficients for the various factors of the hidden sector gauge group. The coefficients c_{im} and l_{im}, which may be found tabulated elsewhere [16], characterize the subgroups of the modular symmetry group $PSL(2, Z)$ that arise for non $T_2 + T_4$ orbifolds [17] i.e. orbifolds for which there are some twisted sectors with fixed planes for which the six-torus T_6 can not be decomposed into a direct sum $T_2 + T_4$ with the fixed plane lying in T_2. In general,

$$
\sum_m c_{im} = 2
\tag{2}
$$

In (1) the moduli T_i are to be understood to include the U moduli associated with Z_2 planes. Factors $H_i(T_i)$ depending on the absolute modular invariant $j(T_i)$ have been introduced. Non-perturbative superpotentials involving $j(T_i)$ may, in principle, arise from orbifold theories containing gauge non-singlet states which become massless at some special values of the moduli [18], though examples are lacking. They may also arise from F-theory compactifications [13]. When the modular symmetry associated with T_i is the full $PSL(2, Z)$, the most general form of $H_i(T_i)$ to avoid singularities in the fundamental domain [18] is

$$H_i(T_i) = (j(T_i) - 1728)^{m_i/2} \; j(T_i)^{n_i/3} P_i(j(T_i)) \tag{3}$$

where m_i and n_i are integers and P_i is a polynomial in j. For generic orbifold compactifications the $H_i(T_i)$ are all 1. Since we do not wish to commit ourselves to any particular choice of gaugino condensates nor of hidden sector matter, we find it convenient to rewrite (1) in the form

$$W_{np} = \Omega(\Sigma) F(T_i) \tag{4}$$

where

$$\Sigma = \; S \; + \sum_{i,m} \delta_i \; c_{im} \; \log(\eta(\frac{T_i}{l_{im}})) \tag{5}$$

and

$$\Omega(\Sigma) = \; \sum_a h_a \; e^{\frac{24\pi^2 \Sigma}{b_a}} \tag{6}$$

with

$$\delta_i = \frac{\delta^i_{GS}}{8\pi^2} \tag{7}$$

and

$$F(T_i) = \prod_{i,m}(\eta(\frac{T_i}{l_{im}}))^{-c_{im}} \prod_j H_j(T_j) \tag{8}$$

In what follows, we shall treat Σ as a parameter to be chosen so that ReS is a approximately 2, and we shall also treat

$$\rho \equiv \frac{\frac{d\Omega}{d\Sigma}}{\Omega} \tag{9}$$

as a free parameter. The parameter ρ is related to the dilaton auxiliary field F_S by

$$\rho = \frac{1 - F_S}{y} \tag{10}$$

with

$$y = \; S + \bar{S} - \sum_i \delta_i \; \log(T_i + \bar{T}_i) \tag{11}$$

If stabilization of the dilaton expectation value involves stringy non-perturbative corrections [10, 11] to the dilaton Kähler potential then we write the dilaton and moduli dependent part of the Kähler potential as

$$K = \; -\sum_i \log(T_i + \bar{T}_i) \; + \; P(y) \tag{12}$$

where $P(y)$ is a function to be determined by non-perturbative string effects. In that case, we shall treat $\frac{dP}{dy}$ and $\frac{d^2P}{dy^2}$, which we shall see occur in the effective potential and soft supersymmetry-breaking terms as free parameters. The form of the effective potential that encompass both possibilities is

$$
\begin{aligned}
V_{eff} &= |W_{np}|^2\, e^{P(y)} \prod_i (T_i + \bar{T}_i)^{-1} \\
&\times \left\{ -3 + \left| \frac{dP}{dy} + \frac{\partial \log W_{np}}{\partial S} \right|^2 \left(\frac{d^2P}{dy^2} \right)^{-1} \right. \\
&\left. + \sum_i (1 + \delta_i \frac{dP}{dy})^{-1} \left| (T_i + \bar{T}_i) \frac{\partial \log W_{np}}{\partial T_i} - 1 + \delta_i \frac{\partial \log W_{np}}{\partial S} \right|^2 \right\}
\end{aligned}
\tag{13}
$$

or, more explicitly,

$$
\begin{aligned}
V_{eff} &= |\Omega(\Sigma)|^2\, |F(T_i)|^2 \prod_j (T_j + \bar{T}_j)^{-1}\, e^{P(y)} \\
&\times \left\{ -3 + \left| \frac{dP}{dy} + \rho \right|^2 \left(\frac{d^2P}{dy^2} \right)^{-1} \right. \\
&\left. + \sum_i (1 + \delta_i \frac{dP}{dy})^{-1} (T_i + \bar{T}_i)^2 \left| (\delta_i \rho - 1)\hat{G}^i + \frac{d \log H_i}{dT_i} \right|^2 \right\}
\end{aligned}
\tag{14}
$$

where

$$
\hat{G}^i = (T_i + \bar{T}_i)^{-1} + \sum_m c_{im} \frac{\partial_{T_i} \eta\left(\frac{T_i}{l_{im}} \right)}{\eta\left(\frac{T_i}{l_{im}} \right)}
\tag{15}
$$

and in the special case of a single condensate, ρ has the value $24\pi^2/b$.

The soft supersymmetry breaking terms may be calculated by standard methods. (See, for example, refs. [19] and [20], from which the earlier literature can be traced.) The gaugino masses are given by

$$
\begin{aligned}
M_a &= m_{3/2}(\mathrm{Re}\, f_a)^{-1} \times \left\{ \frac{\partial \bar{f}_a}{\partial \bar{S}} \left(\frac{d^2P}{dy^2} \right)^{-1} \left(\frac{dP}{dy} + \rho \right) \right. \\
&+ \left. \sum_i \frac{(b_a^{\prime i} - \delta_{GS}^i)}{8\pi^2} \frac{y}{(y - \delta_i)} (\rho \delta_i - 1)^{-1} (T_i + \bar{T}_i)^2 \left| (\rho \delta_i - 1)\hat{G}^i + \frac{d \log H_i}{dT_i} \right|^2 \right\}
\end{aligned}
\tag{16}
$$

where $b_a^{\prime i}$ is the usual coefficient occurring in the string loop threshold corrections to the gauge coupling constants [21, 19] and f_a is the gauge kinetic function. Provided the dilaton expectation value S and its auxiliary field in (16) are real there are no CP violating phases in the gaugino masses. The soft supersymmetry breaking A terms are given by

$$
\begin{aligned}
m_{3/2}^{-1} A_{\alpha\beta\gamma} &= \left(\frac{d^2P}{dy^2} \right)^{-1} \left(\frac{dP}{dy} + \bar{\rho} \right) \frac{dP}{dy} \\
&- \sum_j (1 + \delta_j \frac{dP}{dy})^{-1} (T_j + \bar{T}_j)\left((\delta_j \bar{\rho} - 1)\bar{\hat{G}}^j + \frac{d \log \bar{H}_j}{d\bar{T}_j} \right) \\
&\times \left(1 + n_\alpha^j + n_\beta^j + n_\gamma^j - (T_j + \bar{T}_j) \frac{\partial \log h_{\alpha\beta\gamma}}{\partial T_j} \right)
\end{aligned}
\tag{17}
$$

where the superpotential term for the Yukawa coupling of ϕ_α,ϕ_β and ϕ_γ is $h_{\alpha\beta\gamma}\,\phi_\alpha\phi_\beta\phi_\gamma$, the modular weights of these states associated with modular transformations on the moduli T_i are n_α^i, n_β^i and n_γ^i, and for notational convenience we have included any U moduli as additional T moduli. The usual rescaling by a factor $e^{K/2}\frac{\bar{W}_{np}}{|W_{np}|}$ required to go from the supergravity theory derived from the orbifold compactification of the string theory to the spontaneously broken globally supersymmetric theory has been carried out together with normalization of the scalar fields. (See, for example, ref. [4].) Throughout we shall assume that there is no CP violating phase associated with the dilaton auxiliary field F_S and we shall assume that ρ is a real parameter.

The expression for the soft supersymmetry breaking B term depends on the mechanism adopted for generating the μ term for the Higgs scalars H_1 and H_2, with corresponding superfields ϕ_1 and ϕ_2. If we suppose that the μ term is generated non-perturbatively as an explicit superpotential term $\mu_W\,\phi_1\,\phi_2$ then the B term, which, in this case we denote by B_W is given by

$$
\begin{aligned}
m_{3/2}^{-1}\,B_W \;=\; & -1 + \Big(\frac{d^2 P}{dy^2}\Big)^{-1}\Big(\frac{dP}{dy} + \bar\rho\Big)\Big(\frac{dP}{dy} + \frac{\partial\log\mu_W}{\partial S}\Big) \\
& - \sum_j (1 + \delta_j\frac{dP}{dy})^{-1}\,(T_j + \bar{T}_j)\Big((\delta_j\bar\rho - 1)\bar{\hat{G}}^j + \frac{d\log\bar{H}_j}{d\bar{T}_j}\Big) \\
& \times \;\Big(1 + n_1^j + n_2^j - (T_j + \bar{T}_j)\frac{\partial\log\mu_W}{\partial T_j} - \delta_j\frac{\partial\log\mu_W}{\partial S}\Big)
\end{aligned}
\tag{18}
$$

where n_1^j and n_2^j are the modular weights of the Higgs scalar superfields ϕ_1 and ϕ_2 and again the appropriate rescaling of the Lagrangian has been carried out.

On the other hand, if the μ term is generated by a term of the form $Z\phi_1\phi_2 + (h.c.)$ in the Kähler potential mixing the Higgs superfields [23], then the tree level form of Z is

$$
Z = (T_3 + \bar{T}_3)^{-1}\,(U_3 + \bar{U}_3)^{-1}
\tag{19}
$$

It is assumed that the third complex plane is the Z_2 plane with whose moduli, T_3 and U_3, the untwisted matter fields ϕ_1 and ϕ_2 are associated, for this mechanism. Before rescaling the Lagrangian by $W_{np}/|W_{np}|$ the B term, which in this case we denote by B_Z, is given by

$$
\begin{aligned}
- m_{3/2}^{-1}\mu_Z^{eff}\,B_Z \;=\; & W_{np}\,Z \times \Big\{2 + (T_3 + \bar{T}_3)\Big(\frac{d\log H_3}{dT_3} - \hat{G}^3(T_3, \bar{T}_3) + \text{h.c}\Big) \\
& + (U_3 + \bar{U}_3)\Big(\frac{d\log H_3'}{dU_3} - \hat{G}^3(U_3, \bar{U}_3) + \text{h.c.}\Big) \\
& + \Big[(T_3 + \bar{T}_3)(U_3 + \bar{U}_3)\Big(\frac{d\log\bar{H}_3}{d\bar{T}_3} - \bar{\hat{G}}^3(T_3, \bar{T}_3)\Big)\Big(\frac{d\log H_3'}{dU_3} - \hat{G}^3(U_3, \bar{U}_3)\Big) + \text{h.c}\Big]\Big\} \\
& + W_{np}\,Z \times \Big\{-3 + \Big|\frac{dP}{dy} + \rho\Big|^2 \Big(\frac{d^2 P}{dy^2}\Big)^{-1} \\
& - \sum_i \frac{(T_i + \bar{T}_i)^2}{1 + \delta_i\frac{dP}{dy}}\,\Big|(\delta_i\rho - 1)\hat{G}^i + \frac{d\log H_i}{dT_i}\Big|^2\Big\}
\end{aligned}
\tag{20}
$$

where

$$
\mu_Z^{eff} \;=\; |W_{np}|Z\Big(1 + (T_3 + \bar{T}_3)(\frac{d\log H_3}{dT_3} - \hat{G}^3(T_3, \bar{T}_3))\Big)
$$

$$+ \quad (U_3 + \bar{U}_3)(\frac{d \log H_3^{'}}{dU_3} - \hat{G}^3(U_3, \bar{U}_3))) \tag{21}$$

In (20) and (21), the factors H_i in (8) associated with the moduli T_3 and U_3 have been denoted by $H_3(T_3)$ and $H_3^{'}(U_3)$, respectively. We have also used the fact that the Green-Schwarz parameters δ_i associated with T_3 and U_3 are zero (at least for a pure gauge hidden sector.)The term $Z\phi_1\phi_2 + h.c.$ in the Kähler potential , with Z given in (19), is merely the first of an infinite series arising from the expansion of $\log[1 + \frac{(\phi_1+\bar{\phi}_2)(\phi_2+\bar{\phi}_1)}{(T_1+\bar{T}_2)(U_1+\bar{U}_2)}]$ which *in total* is modular invariant [23]. Because the third plane is a Z_2 plane, the U-modulus is *not* inert under a T-modular transformation:

$$U_3 \to U_3 + \frac{\phi_1\phi_2}{icT_3 + d}$$

Thus although the whole series is modular invariant, it is not modular invariant term by term. It then follows that when we calculate the associated soft supersymmetry-breaking B-term in the effective potential it too is modular invariant, but not term by term. Thus the best we can do at this juncture is to calculate the CP-violating phase of B_Z at a series of minima of V_{eff} related by modular transformations, thereby estimating the scale of CP-violation to be expected in the calculation of observable, and therefore modular invariant, quantities. In section 5, we shall study the simple model of a single overall modulus T with

$$T_1 = T_2 = T_3 = T \tag{22}$$

and T_1, T_2 and T_3 on the same footing in the non-perturbative superpotential W_{np} and the Yukawa couplings $h_{\alpha\beta\gamma}$. In addition, we shall assume an unbroken $PSL(2, Z)$ modular symmetry group for T. In that case, we make the replacements in the above formulae

$$\frac{\partial}{\partial T_i} \to \frac{1}{3}\frac{\partial}{\partial T}, \ \delta_i \to \frac{\tilde{\delta}_{GS}}{3}, \ n_\alpha^i \to n_\alpha, \ \hat{G}^i \to \hat{G} \tag{23}$$

where

$$\hat{G}(T, \bar{T}) = (T + \bar{T})^{-1} + 2\eta^{-1}\frac{d\eta}{dT} \tag{24}$$

If there are non-trivial factors $H_i(T_i)$ in (8) we also make the replacement

$$\frac{dH_i}{dT_i} \to \frac{1}{3}\frac{dH}{dT} \tag{25}$$

The assumption of the Kähler potential mixing mechanism for μ is somewhat unnatural in this simple model because this mechanism requires the presence of a Z_2 plane for the orbifold with an associated U modulus as well as an associated T modulus. If we wish to employ a model in which the supersymmetry breaking is dominated by the T moduli we should then set the auxiliary field for U_3 to zero and to take

$$\hat{G}^3(U_3, \bar{U}_3) - \frac{d \log H_3^{'}}{dU_3} = 0 \tag{26}$$

in (20) and (21).

3 The $Z6 - IIb$ orbifold with multiple gaugino condensates

We shall discuss the case of the $Z6 - IIb$ orbifold in this section and section 4. This example is rich enough to contain a T-modulus (T_1) with associated $PSL(2, Z)$ target-space modular symmetry, and a pair of T- and U- moduli (T_3 and U_3) with the congruence subgroups $\Gamma^0_{T_3}(3)$ and $\Gamma^0_{U_3-2i}(3)$ of the associated target space modular symmetry groups, where

$$\Gamma^0(n) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad ad - bc = 1, \ b = 0 \bmod n \ a, b, c, d \in Z\}$$

If we assume that the dilaton is stabilized by a multiple gaugino condensate scenario the effective potential, is given by the following expression

$$\begin{aligned} V_{eff} &= |\Omega(\Sigma)|^2 |F(T_i)|^2 \prod_i (T_i + \bar{T}_i)^{-1} y^{-1} \Big\{ -3 + |1 - y\rho|^2 \\ &+ \sum_i \frac{y}{(y - \delta_i)} (T_i + \bar{T}_i)^2 \Big| (1 - \delta_i \rho) \hat{G}^i + \frac{d \log H_i}{dT_i} \Big|^2 \Big\} \end{aligned} \tag{27}$$

where

$$F(T_i) = \prod_{i,m} \Big(\eta(\frac{T_i}{l_{im}}) \Big)^{-c_{im}} \prod_j H_j(T_j) \tag{28}$$

and $c_{11} = 2, c_{31} = c_{32} = c_{41} = c_{42} = 1$ and $l_{11} = l_{31} = l_{42} = 1, l_{32} = l_{41} = 3$.

In the case that the j function is not present in the non-perturbative superpotential, one minimizes (27) with respect to the moduli T_i by omitting all the H_i factors. In this case, numerical minimization gives us the following results. Let us start with the T_1 modulus. For $0 < \rho \leq 0.4$ the minimum is at a real value of T_1 (see Fig.2 and Fig.3) which approaches 1 as ρ approaches 0.42. For $0.42 \leq \rho \leq 0.75$, T_1 remains at the fixed point at $T_1 = 1$, and for $\rho \geq 0.75$ the minimum is at the other fixed point at $T_1 = e^{\frac{i\pi}{6}}$. (There are of course also minima at points obtained from these minima by modular transformations. [3]) The other two moduli T_3, U_3 behave as follows. For $0 \leq \rho \leq 0.75$ the minima along the U_3 and T_3 directions in the moduli field space are at the points $\sqrt{3} + i(2 + 3m), \sqrt{3} + i(3n)$ respectively, with $n, m \in Z$. The latter points are zeros of the $\hat{G}^{U_3}$ and $\hat{G}^{T_3}$ functions (See Fig.6). (Again there are also minima obtained from these minima by modular transformations [4] .) For $\rho \geq 0.75$ the minima for both moduli are at their fixed points. More specifically for T_3 the minimum is at $T_3|_{min} = \sqrt{3}/2 + (1.5 + 3n)i$ and for U_3 at $U_3|_{min} = \sqrt{3}/2 + (3.5 + 3m)i$. See also figure 2. Consequently in the region $0 \leq \rho \leq 0.75$ we have anisotropic solutions while for $\rho \geq 0.75$ we have isotropic solutions.

If, as has been discussed above, we allow the possibility that the absolute modular invariant j appears in the non-perturbative superpotential W_{np}, we find complex solutions on the unit circle. For instance for $\rho = 0.5, \delta^{T_1}_{GS} = -10, m_1 = n_1 =$

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[3] This is a non-trivial check of the correct modular invariance properties of V_{eff}.

[4] For example, numerical minimization for $\rho = 0.55$ yields $T_3|_{min} = \sqrt{3}$, but also $T_3|_{min} = 0.4330128 - 0.75i$. The latter point is related to the former through the modular transformation $T_3 \to \frac{T_3}{iT_3+1} \in \Gamma^0(3)$

1, $m_2 = n_2 = m_3 = n_3 = 0$ we obtain (see Fig.4) the following solution

$$
\begin{aligned}
T_3|_{min} &= 1.73205080 + 3m\ i \\
T_1|_{min} &= 0.97097402 \pm 0.23918496\ i \\
U_3|_{min} &= 1.73205080 + (2 + 3n)\ i
\end{aligned}
\tag{29}
$$

The soft A-terms arising from (27) are given by Eq.(17), which reduces to

$$
\begin{aligned}
m_{3/2}^{-1} A_{\alpha\beta\gamma} &= (1 - y\bar{\rho}) \\
&- \sum_j \left(\frac{y}{y - \delta_j}\right)(T_j + \bar{T}_j)\left((\delta_j\bar{\rho} - 1)\bar{\hat{G}}^j + \frac{d\log\bar{H}_j}{d\bar{T}_j}\right) \\
&\times\ \left(1 + n_\alpha^j + n_\beta^j + n_\gamma^j - \frac{\partial\log h_{\alpha\beta\gamma}}{\partial T_j}\right)
\end{aligned}
\tag{30}
$$

in this case. The $\frac{\partial\log h_{\alpha\beta\gamma}}{\partial T_j}$ contribution to the A-terms in (30) is essential for its modular invariance, and can make a significant contribution to any CP-violating phase. Non trivial Yukawa couplings from twisted sector states arise from $\theta\theta\theta^4$ and $\theta^2\theta^2\theta^2$ couplings. We consider both the Yukawa couplings $h(T_1, k = 0)$ where

$$
h(T_1, k) \sim e^{-\frac{2}{3}\pi k^2 T_1}\left[\Theta_3(ikT_1, 2iT_1)\Theta_3(ikT_1, 6iT_1) + \Theta_2(ikT_1, 2iT_1)\Theta_2(ikT_1, 6iT_1)\right]
\tag{31}
$$

which is covariant (invariant) under $T_1 \to T_1 + i$, and the linear combination of Yukawas

$$
h(T_1, k = 0) + (\pm\sqrt{3} - 1)h(T_1, k = 1)
\tag{32}
$$

which is covariant under $T_1 \to \frac{1}{T_1}$ (see the discussion in §5.)

Similarly the soft susy-breaking B_W-term, arising from (27) is given by (18), which reduces to

$$
\begin{aligned}
m_{3/2}^{-1} B_W &= -1 + (-1 + y\bar{\rho})(-1 + y\frac{\partial\log\mu_W}{\partial S}) \\
&- \sum_j \left(\frac{y}{y - \delta_j}\right)(T_j + \bar{T}_j)\left((\delta_j\bar{\rho} - 1)\bar{\hat{G}}^j + \frac{d\log\bar{H}_j}{d\bar{T}_j}\right) \\
&\times\ \left(1 + n_1^j + n_2^j - (T_j + \bar{T}_j)\frac{\partial\log\mu_W}{\partial T_j} - \delta_j\frac{\partial\log\mu_W}{\partial S}\right)
\end{aligned}
\tag{33}
$$

in this case, where [23] μ_W is given by

$$
\mu_W \propto W_{np}\frac{\partial\log\eta(T_3)\eta(\frac{T_3}{3})}{\partial T_3}\frac{\partial\log\eta(U_3)\eta(\frac{U_3}{3})}{\partial U_3}
\tag{34}
$$

Although this form of μ_W has the correct behaviour under T_3- and U_3-modular transformation, the resulting expression for B_W is not modular invariant; (the reason for this is that exact modular invariance requires the use of the one-loop corrected Kähler potential [23].)

The values given above for the moduli T_i at the minimum of V_{eff}, when the Dedekind η function is the only modular function present in W_{np}, have rather striking consequences for the possible CP violating phases in the soft supersymmetry-breaking terms. It might have been thought *a priori* that when T_i is at a fixed point at $e^{\frac{i\pi}{6}}$ a

CP violating phase of order 10^{-1} might be induced. However, as has been observed earlier [5], if T_i is precisely at a fixed point value, and/or at a zero of $\hat{G}(T_i, \bar{T}_i)$, the CP violating phase vanishes identically, as can be seen from (16)-(20). To illustrate the importance of the latter statement we have plotted the dependence of the imaginary part of the Eisenstein function $\hat{G}^{T_3}$ of the T_3 modulus, with respect to the imaginary part of the modulus, for fixed real part (see Fig. 5) of the latter. We note from the graphs that if the moduli are exactly at a fixed point, at the minimum of the potential energy, zero phases occur while even a small departure from the imaginary part at $ImT_3 = 1.5$ in our example, can cause an imaginary part for $\hat{G}^{T_3}$ of order 10^{-2} which can be fed to the soft-supersymmetry breaking terms as a phase of order 10^{-2}. Later, when we discuss the model in which non-perturbative corrections to the Kähler potential are responsible for the dilaton stabilization, we will see that there is another mechanism for suppressing CP phases which is due to the very rapid variation of the imaginary part of $\hat{G}^i$ with ReT as Re T moves away from 1 if Im T is held fixed.

For the multiple gaugino condensate scenario in the $Z6 - IIb$ orbifold we conclude the following: If the Dedekind η function is the *only modular form* appearing in the non-perturbative superpotential there are no CP violating phases in the soft-supersymmetry breaking terms.

If, on the other hand, the j function is also involved in W_{np} besides the η function, then CP violating phases may arise. For the Yukawa couplings we have considered, and for the particular solution presented in Eq.(29), we obtain CP violating phases not greater than 2×10^{-5} in the A term. However, the B_W soft superymmetry-breaking terms lead to a zero phase for the solution presented in (29). Choi [24] has also obtained small CP- violating phases in a class of multiple gaugino condensate models which possess an approximate Peccei-Quinn symmetry.

Thus overall, in the multiple gaugino condensate scenario in the $Z6 - IIb$ orbifold, we conclude that if both the j-function and the Dedekind η function are present in W_{np} then CP-violating phases not greater than 10^{-5} arise.

4   The $Z6-IIb$ orbifold model with non-perturbative corrections to the dilaton Kähler potential

In this section we continue the discussion of the $Z6 - IIb$ orbifold introduced in section 3. If the dilaton VEV is stabilised by non-perturbative corrections to the Kähler potential, we minimize the effective potential with the modular invariant y fixed at 4, for different values of the parameters $\frac{dP}{dy}$ and $\frac{d^2P}{dy^2}$. The effective potential in this case is given by the following expression

$$
\begin{aligned}
V_{eff} &= |\Omega(S)|^2 |F(T_i)|^2 \prod_i (T_i + \bar{T}_i)^{-1} e^{P(y)} \\
&\times \left\{ -3 + \left| \frac{dP}{dy} + \frac{24\pi^2}{b} \right|^2 \left(\frac{d^2P}{dy^2} \right)^{-1} \right. \\
&+ \left. \sum_i (1 + \delta_i \frac{dP}{dy})^{-1} (T_i + \bar{T}_i)^2 \left| (\delta_i \frac{24\pi^2}{b} - 1)\hat{G}^i + \frac{d \log H_i}{dT_i} \right|^2 \right\}
\end{aligned}
$$ (35)

with

$$
\Omega(S) = h \, e^{24\pi^2 S/b}
$$ (36)

and

$$F(T_i) = \prod_{i,m} \left(\eta \left(\frac{T_i}{l_{im}} \right) \right)^{-c_{im}(1 - \frac{3\delta_{GS}^i}{b})} \prod_j H_j(T_j) \tag{37}$$

and $c_{11} = 2, c_{31} = c_{32} = c_{41} = c_{42} = 1$ and $l_{11} = l_{31} = l_{42} = 1, l_{32} = l_{41} = 3$.

In this case we obtain the following results. First, if the non-perturbative superpotential W_{np} includes only the Dedekind eta function, the values of the T_1 modulus for a wide range of the parameters are either real or at a fixed point of $PSL(2, Z)$. On the other hand, the values at the minimum of the effective potential for the T_3 and U_3 moduli are either at the points $\sqrt{3} + i3p$, $\sqrt{3} + i(2 + 3m)$ (p, m are integers) respectively, or they are at the fixed points of $\Gamma^{(0)}(3)$.

Second, if we allow for the possibility that the j function appears in the form of the non-perturbative superpotential as well as the η function, we obtain solutions at complex values of the moduli on the *unit circle*. For instance for $\frac{dP}{dy} = -1/4$, $\frac{d^2P}{dy^2} = 0.6$, $m_1 = n_1 = 1, m_i = n_i = 0$ for $i = 2, 3$ and $\tilde{\delta}_{GS}^{T_1} = -10$ (see fig.8) the following solution occurs

$$\begin{aligned}
T_1|_{min} &= 0.97090183 \pm 0.23947783 \ i \\
T_3|_{min} &= 0.8660254 + 1.49999999 \ i \\
U_3|_{min} &= 0.8660254 + 3.5000000 \ i
\end{aligned} \tag{38}$$

In this case, the minimum on the unit circle for the T_1 modulus is at a zero of the quantity

$$(\delta_i \frac{24\pi^2}{b} - 1)\hat{G}^i + \frac{d \log H_i}{dT_i} \tag{39}$$

appearing in (35). This can be demonstrated analytically and agrees with the numerical analysis. In consequence, as we will discuss below, the CP-violating phases are zero for minima of the V_{eff} which are zeros of (39).

The soft terms arising from (35) are given by

$$\begin{aligned}
m_{3/2}^{-1} A_{\alpha\beta\gamma} &= \left(\frac{d^2P}{dy^2} \right)^{-1} \left(\frac{dP}{dy} + \frac{24\pi^2}{b} \right) \frac{dP}{dy} \\
&\quad - \sum_j \left(1 + \delta_j \frac{dP}{dy} \right)^{-1} (T_j + \bar{T}_j) \left((\frac{3\delta_{GS}^j}{b} - 1)\bar{\hat{G}}^j + \frac{d \log \bar{H}_j}{d\bar{T}_j} \right) \\
&\quad \times \left(1 + n_\alpha^j + n_\beta^j + n_\gamma^j - (T_j + \bar{T}_j) \frac{\partial \log h_{\alpha\beta\gamma}}{\partial T_j} \right)
\end{aligned} \tag{40}$$

If the Dedekind η function is the only modular function that appears in W_{np}, we find that the CP-violating phases are either zero or much smaller than 10^{-3}.

If the j-function is present in W_{np}, together with the Dedekind η function, and we use solutions such as (38), then the CP-violating phases are zero since, as noted above, these complex minima are zeros of(39). However, for different values of $\frac{dP}{dy}$ and $\frac{d^2P}{dy^2}$ there are other minima where significant CP-violating phases arise. For example, for $\frac{dP}{dy} = -2.45, \frac{d^2P}{dy^2} = -0.45, m(T_1) = n(T_1) = 1, m(T_3) = n(T_3) = n(U_3) = m(U_3) = 0, \delta_{GS}^{T_1} = -20$ we obtain the following solution (see Fig.7)

$$\begin{aligned}
T_3|_{min} &= 0.31054252 + \ 2.999998 \ i \\
T_1|_{min} &= 0.97084106 \pm 0.23972408 \ i \\
U_3|_{min} &= 0.07838707 + 1.84680549 \ i
\end{aligned} \tag{41}$$

This solution leads to a CP-violating phase $\phi(B_W)$ of order 10^{-1}. On the other hand $\phi(A)$ is of order 10^{-2}.

5 The single overall modulus case

In this case the relevant formulae for the effective potential and soft supersymmetry-breaking terms are obtained by making the substitutions (23)-(25) (See also [25].

The $\frac{\partial \log h_{\alpha\beta\gamma}}{\partial T}$ contribution to the A-terms deriving from (17) is essential for its modular invariance and can make a significant contribution to any CP violating phase. For illustrative purposes we have taken $h_{\alpha\beta\gamma}$ to be of the form encountered [22] when each of the states ϕ_α, ϕ_β and ϕ_γ is in the particular twisted sector of the $Z_3 \times Z_6$ orbifold with the same twisted boundary conditions as the twisted sector of the Z_3 orbifold. This is an appropriate choice because the $Z_3 \times Z_6$ orbifold has three $N = 2$ moduli, T_i, $i = 1, 2, 3$, so that the model of a single overall modulus $T = T_1 = T_2 = T_3$ is consistent. In this case, if we arrange $h_{\alpha\beta\gamma}$ to be covariant under the $T \to \frac{1}{T}$ modular transformation, it is a product of 3 factors, one for each complex plane, of the form

$$h(T_i, k_i = 0) + (\pm\sqrt{3} - 1)h(T_i, k_i = 1) \tag{42}$$

where

$$h(T_i, k_i) \sim e^{-\frac{2}{3}\pi k_i^2 T_i}\left[\Theta_3(ik_i T_i, 2i T_i)\Theta_3(ik_i T_i, 6i T_i) + \Theta_2(ik_i T_i, 2i T_i)\Theta_2(ik_i T_i, 6i T_i)\right] \tag{43}$$

Each of the modular weights n_α, n_β and n_γ has the value -2. Yukawa couplings associated with untwisted matter fields have modular weight 0 and are generically T-independent constants.

In the case of multiple gaugino condensates with perturbative Kähler potential, minimization of the effective potential at fixed Σ for different real values of the parameter ρ with Re S taken to be about 2 leads to the following conlusions. It can be seen analytically that the fixed points of $PSL(2, Z)$ at $T = 1$ and $T = e^{\frac{i\pi}{6}}$, at which $\hat{G}(T, \bar{T})$ is zero, are always extrema (even for $\tilde{\delta}_{GS} \neq 0$.) For $0.1 \leq \rho \leq 0.4$, the minimum is at a real value of T which approaches 1 as ρ approaches 0.42. For $0.42 \leq \rho \leq 0.75$, T remains at the fixed point at $T = 1$, and for $\rho \geq 0.8$ the minimum is at the other fixed point at $T = e^{\frac{i\pi}{6}}$ (See Fig.9). (There are of course also minima at points obtained from these minima by modular transformation.) This resembles what happens for a single condensate but treating the dilaton auxiliary field F_S as a free parameter to simulate dynamics stabilizing the dilaton expectation value [26].

Gaugino condensate models (with perturbative Kähler potential) in general have negative vacuum energy at the minimum. However, as other authors have emphasized [26], the solution to the vanishing cosmological constant problem is probably in the realm of quantum gravity and as the present type of discussion treats gravity classically we need not necessarily impose vanishing vacuum energy as a constraint on the theory. On the other hand, if we do arrange for zero vacuum energy by introducing an extra matter field which does not mix with the dilaton and moduli fields [4] then the effect in the minimization of the effective potential with respect to T is that the factor premultiplying the bracket in (14) is not to be differentiated. Then, for $0.25 \leq \rho \leq 2.15$ minima occur at the fixed points at $T = 1$ and $T = e^{\frac{i\pi}{6}}$. For $\rho \geq 2.2$ there is a single real minimum.

In the case of a single gaugino condensate, but with the dilaton expectation value being stabilized by stringy non-perturbative corrections to the dilaton Kähler potential, minimization of the effective potential with y fixed at 4 for different values of the parameters $\frac{dP}{dy}$ and $\frac{d^2P}{dy^2}$ leads instead to the following outcome. For a wide range of choices of these parameters T is either at a fixed point value or it is real. However, for $\frac{dP}{dy} = -11/4$ and $\frac{d^2P}{dy^2} = -1.3$, one obtains

$$T|_{min} = 5.234339 + 0.0009575i \qquad (44)$$

and the potential is more flat.

The consequences of the values of the modulus T at the minimum for possible CP violating phases in the soft supersymmetry breaking terms are similar to those of the $Z6 - IIb$ orbifold Thus, for the case where the dilaton is stabilized by a multiple gaugino condensate with perturbative Kähler potential, there are no CP violating phases in the soft supersymmetry breaking terms. In the case of a single gaugino condensate with the dilaton stabilized by non-perturbative corrections to the dilaton Kähler potential, the conclusion is the same for a wide range of values of $\frac{dP}{dy}$ and $\frac{d^2P}{dy^2}$. However, in this case, it is possible for the minimum for T to be at a complex value away from the fixed point as, for example, in (44). At first sight, there might then be a CP violating phase of order 10^{-3}. However, the CP violating phases are far smaller than this (of order 10^{-15} for T as in (44).) The reason for this is the very rapid variation of the imaginary part of $\hat{G}(T, \bar{T})$ with ReT as Re T moves away from 1 if Im T is held fixed (see fig. 10). The imaginary part of $\hat{G}(T, \bar{T})$ varies by 11 orders of magnitude as Re T goes from $\frac{\sqrt{3}}{2}$ to 5.0. The possibility of suppressing CP violating phase in this way has been suggested earlier [5] in the context of orbifold models with broken $PSL(2, Z)$ modular symmetries.

If we allow the possibility that the non-perturbative superpotential W_{np} involves the absolute modular invariant $j(T)$ as well as the Dedekind eta function [18] then the situation is very different. Then, W_{np} contains an extra factor $H(T)$ where the most general form of $H(T)$ to avoid singularities inside the fundamental domain [18] is

$$H(T) = (j - 1728)^{m/2} j^{n/3} P(j) \qquad (45)$$

where m andf n are integers and $P(j)$ is a polynomial in j. It is then possible, for some choices of H to obtain (complex) minima of the effective potential for T that lead to CP violating phases in the soft supersymmetry breaking terms of order $10^{-4} - 10^{-1}$. Let us start with the case of stabilizing the dilaton by multiple gaugino condensate, then for $P(j) = 1$ and $m = n = 1$, $\tilde{\delta}_{GS} = -\frac{30}{8\pi^2}$, $\rho = 0.45$, we find that the minimum is on the unit circle at

$$T|_{min} = 0.971353713 \pm 0.2376383050i \qquad (46)$$

For the Yukawa couplings which we have considered (see below), this leads to a CP violating phase not greater than 10^{-4}. Also for $P(j) = 1$ and $m = n = 1$, $\tilde{\delta}_{GS} = -\frac{50}{8\pi^2}$, $\rho = 0.26$ the minimum is also on the unit circle at

$$T|_{min} = 0.971352323 \pm 0.237643985i \qquad (47)$$

which again leads to CP violating phase not greater than 10^{-4} in the A term. On the other hand, if we assume that the dilaton is stabilized by non-perturbative corrections

to the Kähler potential, then it is possible to find minima of the effective potential at complex values of the $T-$ modulus not only on the boundary of the fundamental domain *but also inside the fundamental domain*. In this case larger CP violating phases arise.

Let us start with the solutions on the unit circle. For $m = n = 1$ we can obtain for a wide range of parameters for $\frac{dP}{dy}$ and $\frac{d^2P}{dy^2}$ solutions on the unit circle. For instance a representative solution is:

$$T|_{min} = 0.971143367 \pm 0.238496458\,i \tag{48}$$

Where $\frac{dP}{dy} = 2.45$ and $\frac{d^2P}{dy^2} = 0.45$ is our choice for our non-perturbative parameters. The CP-violating phases in this case are zero because as we argued in §4 these solutions correspond to zeros of Eq.(39). We can also obtain local minima on the unit circle for the following choice of $(m, n) = (1, 3)$. Let us a give an example:

$$T|_{min} = 0.98650708 \pm 0.163718557\,\,i \tag{49}$$

for $\frac{dP}{dy} = -1.15$ and $\frac{d^2P}{dy^2} = 0.45$ In this case however and in general for $n \geq 2$ the $T = e^{\frac{i\pi}{6}}$ is a minimum with zero cosmological constant.

Let us now describe , solutions not on the boundary of the fundamental domain but inside where it is possible for some values of the parameters $\frac{dP}{dy}$ and $\frac{d^2P}{dy^2}$ to obtain phases that exceed the current experimental limit. As a result we can constrain our non-perturbative parameter space. For $\frac{dP}{dy} = -1.5$ and $\frac{d^2P}{dy^2} = -0.2$, $m = 1, n = 3, \delta_{GS} = -30$ we obtain the following global minima :

$$T|_{min} = 1.01196232 + 0.16800043\,i \tag{50}$$

and its $T - dual$ under the generator $T \rightarrow \frac{1}{T}$,

$$T|_{min} = 0.96167455 - 0.15965193\,i \tag{51}$$

Of course as we said before we can obtain minima connected to the above by modular transformations as well. [5]

Both minima lead to a phase $\phi(A)$ of order 10^{-2}. The foregoing results need a little amplification. For minima connected by $T \rightarrow T + i$ the Yukawa $h_{\alpha\beta\gamma} = h(T, k = 0)$ leads to the same CP violating phases at both minima, while for minima connected by $T \rightarrow \frac{1}{T}$ the Yukawa $h_{\alpha\beta\gamma} = h(T, k = 0) + (\sqrt{3} - 1)h(T, k = 1)$, which transforms as $h_{\alpha\beta\gamma}(1/T) = Th_{\alpha\beta\gamma}(T)$, also leads to the same CP violating phases at both minima. Both are of order 10^{-2}. However, since there is no linear combination of Yukawas which has modular weight 1 with respect to *all* modular transformations, we cannot do better than characterize the scale of the CP violating phases in this way. It is important to note that since V_{eff} is modular invariant, the calculation of the electric dipole moment of the neutron, for example, will necessarily yield a modular invariant result, presumably with magnitude characteristic of the order 10^{-2} scale of the CP violating phase of $A_{\alpha\beta\gamma}$. This calculation will necessarily entail contributions from more than one A term. For the B_W soft term we similarly obtain a phase at both of the minima given in Eq.(50)-(51) of order 10^{-3}.

---

[5] Again the appearance of minima connected by modular transformations is a non-trivial check of the correct implementation of modular invariance properties in our expression for V_{eff}.

Similarly, for $\frac{dP}{dy} = -1.4, \frac{d^2P}{dy^2} = -0.1, \ m = 1, n = 3, \delta_{GS} = -30$ (see Fig. 11) we obtain

$$T|_{min} = 0.79314323 + 0.11307387 \, i \tag{52}$$

its $T - dual$ under $T \to \frac{1}{T}$

$$T|_{min} = 1.23569142 - 0.17616545 \, i \tag{53}$$

as well as the $T - dual$ of the later under $T \to T + i$

$$T|_{min} = 1.23569142 + 0.82383457 \, i \tag{54}$$

At the above three points of the moduli space we find a phase $\phi(A)$ of order $10^{-3} - 10^{-2}$. On the other hand the CP violating phase $\phi(B_W)$ and $\phi(B_Z)$ are of order $10^{-2} - 10^{-1}$ at the three points of the moduli space.

For $m = n = 1, \delta_{GS} = -30, \frac{dP}{dy} = -2.45, \frac{d^2P}{dy^2} = -0.45$ we get the following representative solutions (see Fig. 12):

$$T|_{min} = 0.6649428 + 0.2387706 \, i \tag{55}$$

its $T - dual$ under $T \to \frac{1}{T}$

$$T|_{min} = 1.332122 - 0.4783445 \, i \tag{56}$$

T as well as the $T - dual$ of the later under $T \to T + i$

$$T|_{min} = 1.332122 + 0.5216555 \, i \tag{57}$$

In this case at the 3 points of the moduli space we obtain a phase $\phi(A)$ of order 10^{-1}. On the other hand $\phi(B_W)$ is of order $10^{-2} - 10^{-1}$ in the above points of the moduli space. Also $\phi(B_Z)$ is of order $10^{-3} - 10^{-1}$.

Let us summarize the dependence of the minima on the integers m, n. For $m = 0, n = 3, T = e^{\frac{i\pi}{6}}$ is a minimum with $V = 0$. The other fixed point $T = 1$ is a minimum with either positive or negative energy dependening on the size and sign of the stringy non-perturbative parameters $\frac{dP}{dy}, \frac{d^2P}{dy^2}$. We also get real minima of $O(1)$. For $m = n = 2$ both fixed points are mimima with zero vacuum energy. We also have solutions on the unit circle with negative or positive energy depending on the stringy parameters. For $m = 0, n = 2 \ T = e^{\frac{i\pi}{6}}$ is a minimum with $V = 0$. Again the other fixed point $T = 1$ is a minimum with either positive or negative energy depending on the size and sign of the stringy non-perturbative parameters $\frac{dP}{dy}, \frac{d^2P}{dy^2}$. For $m = 2, n = 0 \ T = 1$, is a zero V minimum. We also get solutions on the unit circle. For $m = 1, n = 2, T = e^{\frac{i\pi}{6}}$ is a zero energy minimum. Solutions on the unit circle with are also obtained. However, in this case we also get minima with negative energy inside the fundamental domain of the T-modulus (see Fig. 1). For instance, for $\frac{dP}{dy} = -1.5, \frac{d^2P}{dy^2} = -0.15$ we get

$$
\begin{aligned}
T|_{min} &= 1.2819602 + 0.2350069 \, i \\
T|_{min}(1/T) &= 0.7546934 - 0.13834922 \, i
\end{aligned}
\tag{58}
$$

For this particular solution both A and B soft supersymmetry-breaking terms lead to a phase of order 10^{-1} at both points of the moduli space. For $m = 1, n = 3$ as we

saw above we get global minima inside the fundamental domain, zero energy minima at $T = e^{\frac{i\pi}{6}}$, plus solutions on the unit circle. For $m = 1, n = 1$ solutions inside the fundamental domain plus solutions on the unit circle. For $m = 3, n = 0$, zero energy minima at $T = 1$, and solutions on the boundary of the fundamental domain of the form $T|_{min} = O(1) + 0.5\, i$ together with their $T-$ duals.

In conclusion, whether the dilaton expectation value is stabilized by a multiple gaugino condensate or by stringy corrections to the dilaton Kähler potential, we have found that , provided the superpotential does not contain the absolute modular invariant $j((T)$, CP violating phases in the soft supersymmetry-breaking terms are either zero or much smaller than 10^{-3}. Zero phases occur when the minimum for the modulus T is at a zero of $\hat{G}(T, \bar{T})$ or is real. Phases much smaller than 10^{-3} occur with T at a complex value with the real part of T far from its value at a zero of $\hat{G}(T, \bar{T})$, because of the rapid variation of the imaginary part of $\hat{G}(T, \bar{T})$ as $Re\, T$ varies. However, if we allow the more general possibility that W_{np} involves $j(T)$ as well as $\eta(T)$, as may arise from orbifold theories if the theory contains gauge non-singlet states that are zero at some special values of the moduli [18] or may arise from F-theory compactifications [13], then it is possible in some models to obtain CP violating phases in the soft supersymmetry breaking terms of order $10^{-4} - 10^{-1}$. The largest phases occur for minima of the potential inside the fundamental domain of the $PSL(2, Z)\, T-$ modulus.

6   Conclusions

In this work we studied the CP-violating properties of soft supersymmetry-breaking terms emerging from target space modular invariant effective string supergravities and in which supersymmetry is broken by non-perturbative corrections to the dilaton Kähler potential (stringy-effects) and gaugino-condensation (field theoretic non-perturbative effects). We found that the CP properties depend on the properties of the modular functions involved as well as on the mechanism for stabilizing the dilaton. The following remarkable picture emerges. In the case of racetrack models (multiple gaugino condensates) with modular symmetries which are subgroups of $PSL(2, Z)$ as well as with the full $PSL(2, Z)$ modular symmetry (overall modulus case), if the Dedekind η function is the *only modular form* appearing in the W_{np} there are no CP-violating phases in the resulting soft supersymmetry-breaking terms. This is the case because minimization of the effective potential, V_{eff}, results in either real values for the orbifold moduli T_i or complex values at the fixed points of the duality group. Complex values at the duality group fixed points correspond to zeros of the Eisenstein functions $\hat{G}^i$ appearing in the expressions for the soft susy-breaking terms. As a result, in both cases the soft supersymmetry-breaking terms *are real*. In the case that the dilaton is stabilized by non-perturbative stringy corrections to the Kähler potential, with the η function again the only modular function appearing in W_{np}, the resulting CP phases are either zero or much smaller than of order 10^{-3}.

The strong experimental upper bound on the electric dipole moment of the neutron translates into bounds on the CP-violating phases of the soft supersymmetry-breaking A and B terms $\phi(A), \phi(B) \leq O(10^{-3})$. The results we have obtained afford the possibility of a stringy explanation of these bounds. As we emphasized in the introduction, the dipole moment tends to be two to three orders of magnitude too large in generic supersymmetric models.

In the case that the absolute modular invariant function j appears together with the η function in W_{np}, and in the case of racetrack models, for a large region of the parameter space the CP phases are suppressed below the experimental limit even for complex values of the moduli T_i on the unit circle at the minimum, and in some regions are close to the current experimental limit;(we find phases less than about 10^{-4}). In the case, that the dilaton is stabilized by non-perturbative stringy corrections to the Kähler potential, then in some regions of the parameter space larger CP-violating phases can arise, of order $10^{-3} - 10^{-1}$ for values of the moduli *inside the fundamental domain* of the modular group. Our solutions at the minimum of the effective potential at complex values for the orbifold moduli T_i inside the fundamental domain, provide a counterexample to the conjecture of [18] that the minima of the target space duality invariant potential occur only on the boundary of the fundamental domain of the modular group. Therefore, in this case we obtain constraints on the non-perturbative Kähler potential parameters from resulting phases that exceed the current experimental limits. However, we must emphasize again that even in the above case, there exist CP-violating phases in the soft-supersymmetry breaking terms , which have the correct order of magnitude for *explaining* [27] the severe experimental limits coming from the electic dipole moment of the neutron and, potentially , the tiny observed CP violation [1].

Our work, motivates the investigation [28] of other duality invariant string theories in which modular functions of higher genus are involved. As has been shown in [29], genus-2 theta functions appearing in the threshold corrections to the gauge couplings which include dependence on the continous Wilson line moduli , can explain the discrepancy of the string scale unification with the observed scale coming from extrapolation of the LEP results. The values of the moduli in this case are close to the ones favoured by duality invariant gaugino condensates. Also, the study of CP-violation in the Kobayashi-Maskawa matrix in this class of models deserves investigation [28]. As we saw the Yukawa couplings for twisted matter fields transform non-trivially under the modular group and can therefore develop complex phases.

7 Acknowledgments

A Modular functions

We list here some useful formulae for the modular functions involved in our work. We first discuss the case of the full modular symmetry $PSL(2, Z)$.

A modular form of weight k is a holomorphic function f,that under a generic modular transformation

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad a, b, c, d \in Z, \quad ad - bc = 1 \tag{59}$$

transforms as

$$f\left(\frac{aT - ib}{icT + d}\right) = (icT + d)^k f(T) \tag{60}$$

Typical modular forms are the *Eisenstein series* in terms of which the $\eta(T)$ and $j(T)$ functions can be represented. Thus an Eisenstein series is a modular form of weight k given by the expression

$$G_k(T) = 2\zeta(k)\left(1 - \frac{2k}{B_k}\sum_{n\geq 1}\sigma_{k-1}(n)\,e^{-2\pi nT}\right) \tag{61}$$

with $\zeta(k)$ is Riemann's zeta function, B_k are the Bernoulli numbers and $\sigma_p(n)$ is the sum of the p-powers of all divisors of n.

The absolute discriminant Δ is a modular form of weight 12 and is given by

$$\Delta = \frac{675}{256\pi^{12}}\left(20G_4^3 - 49G_6^2\right) \tag{62}$$

while the absolute modular invariant j is given by the equation

$$j = \frac{3^6 5^3}{\pi^{12}}\frac{G_4^3}{\Delta} \tag{63}$$

By a theorem, proven in [30], every modular form $f(T)$, regular in the fundamental domain, can be written as follows:

$$f(T) = (G_6(T))^s (G_4(T))^t (\eta(T))^{2r-12s-8t} P(J) \tag{64}$$

For every modular form f, of weight k, the following derivative

$$D_T f(T) = f'(T) + \frac{k}{2\pi}G_2(T)f(T) \tag{65}$$

is a modular form of weight $k+2$. This can be proven by using the definition of a modular form and the relation of G_2 with the logarithmic derivative of the η function.
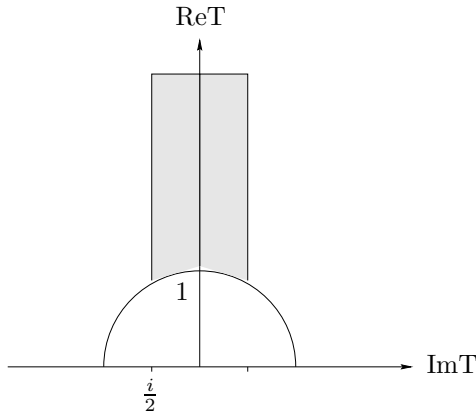
Figure 1: Fundamental domain Γ of the full modular group

The congruence subgroup $\Gamma^0(3)$ has as generators the transformations $T \to T + 3\,i$, $T \to \frac{T}{iT+1}, I$.

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Figure 2: Behaviour of the real parts of T_1 and T_3 moduli with respect to the parameter ρ in multiple gaugino condensate models

Figure 3: Dependence of the imaginary parts of T_1 and T_3 moduli with ρ as in Figure 2.

Figure 4: Solutions on the unit circle in the multiple gaugino condensate case.

Figure 5: Dependence of $Im\hat{G}^{T_3}$ with respect to ImT_3, with the real part of T_3 fixed at $e^{\frac{i\pi}{6}}$ upper figure, and at 1.3 lower figure.

Figure 6: Zero of $\hat{G}^{T_3}$ other than the fixed point.

Figure 7: Dependence of V_{eff} on the U_3 modulus for fixed values of the other moduli at their minimum see (41).

Figure 8: The solution on the unit circle for the T_1 modulus

Figure 9: Variation of ReT at the minimum of V_{eff} with ρ, in multiple gaugino condensate models.

Figure 10: Variation of the logarithm of $Im\hat{G}$ with respect to ReT.

Figure 11: The effective potential for the non-perturbative dilaton Kähler potential in the overall modulus case.

Figure 12: Solutions of V_{eff} inside the fundamental domain with $m = n = 1$, in the overall single modulus case.
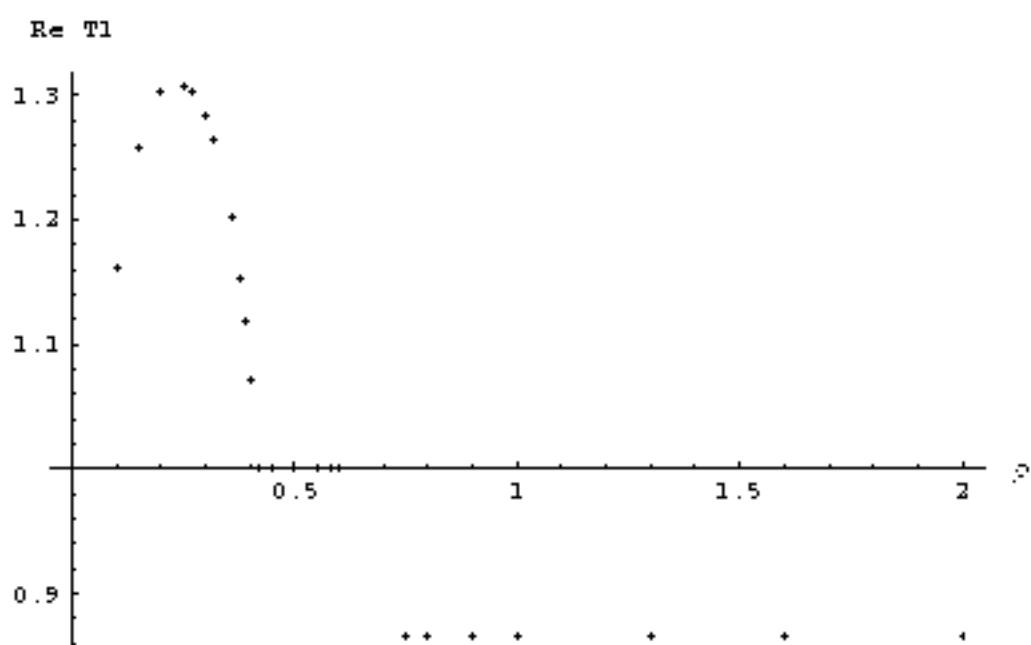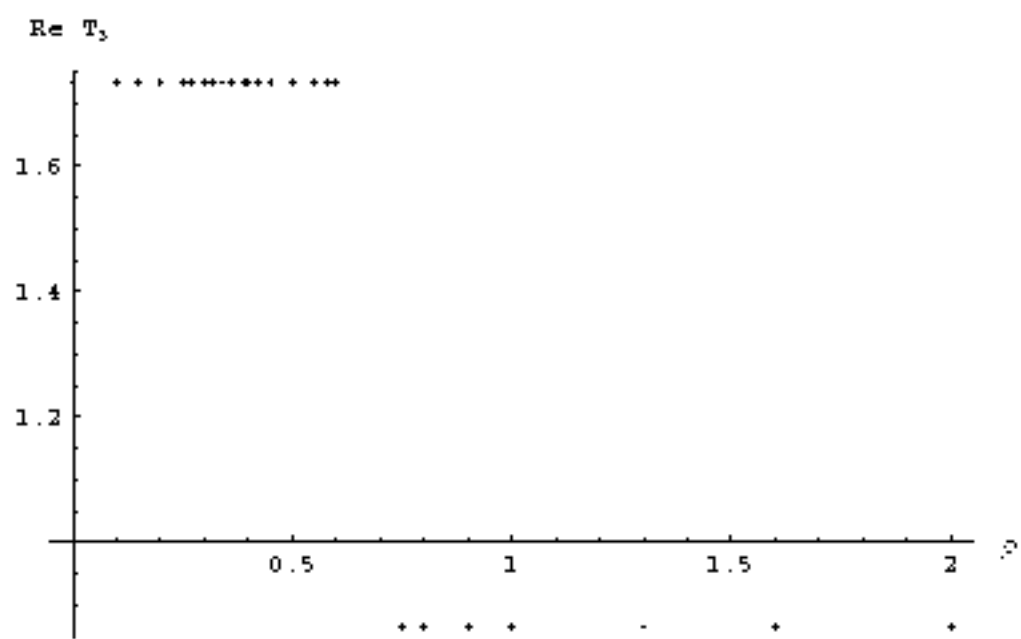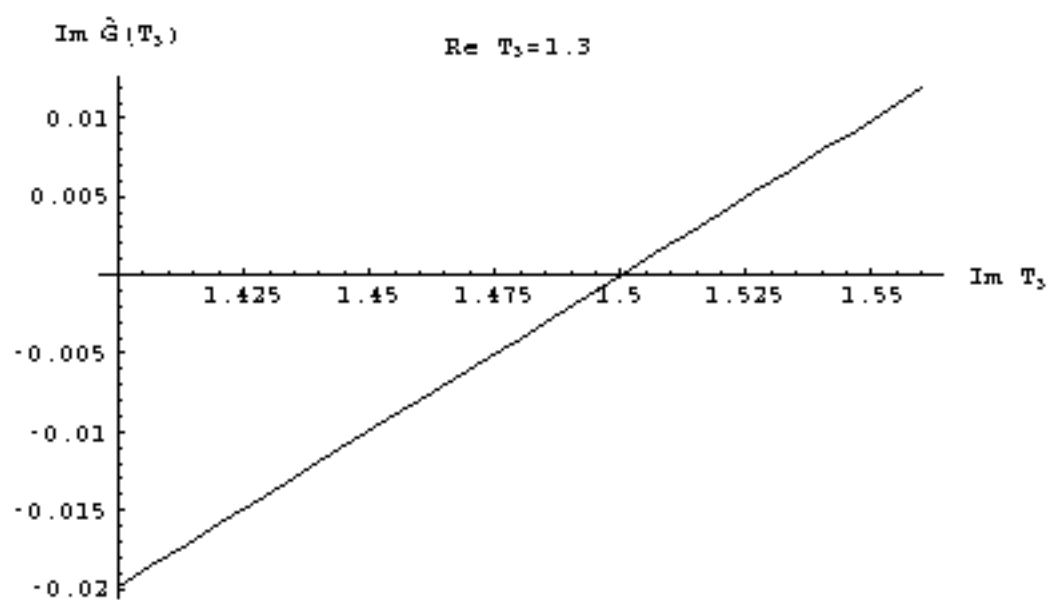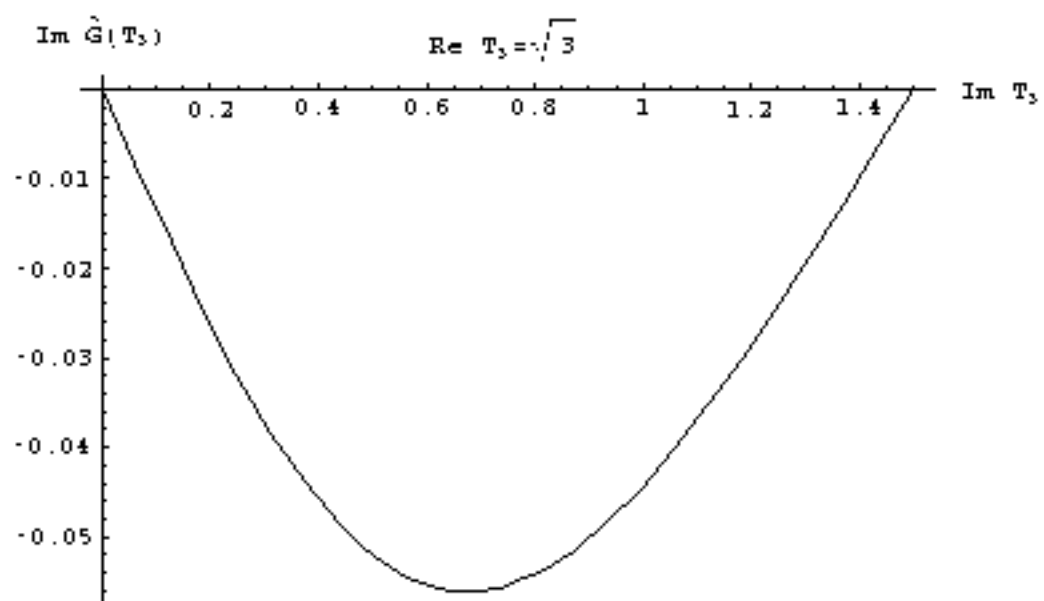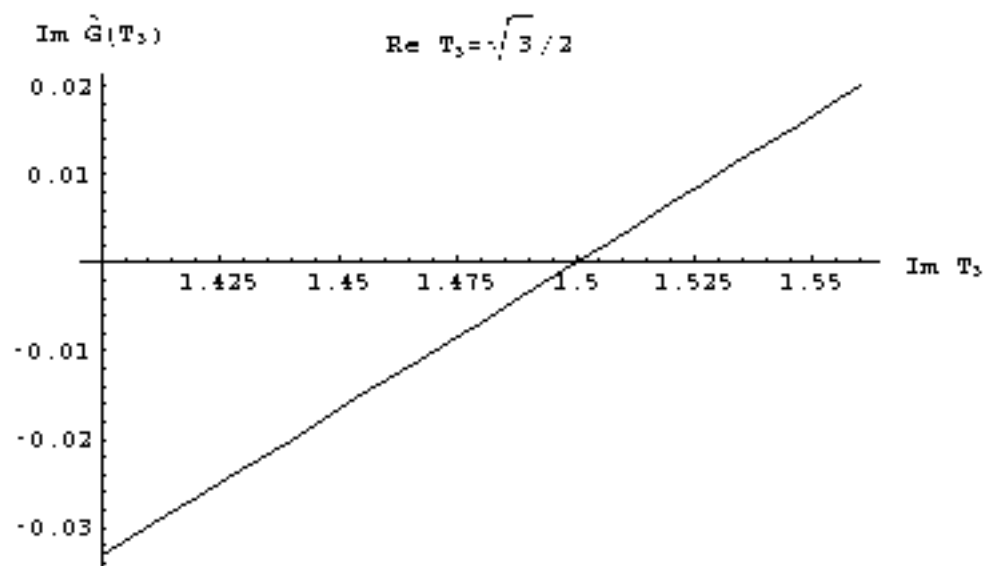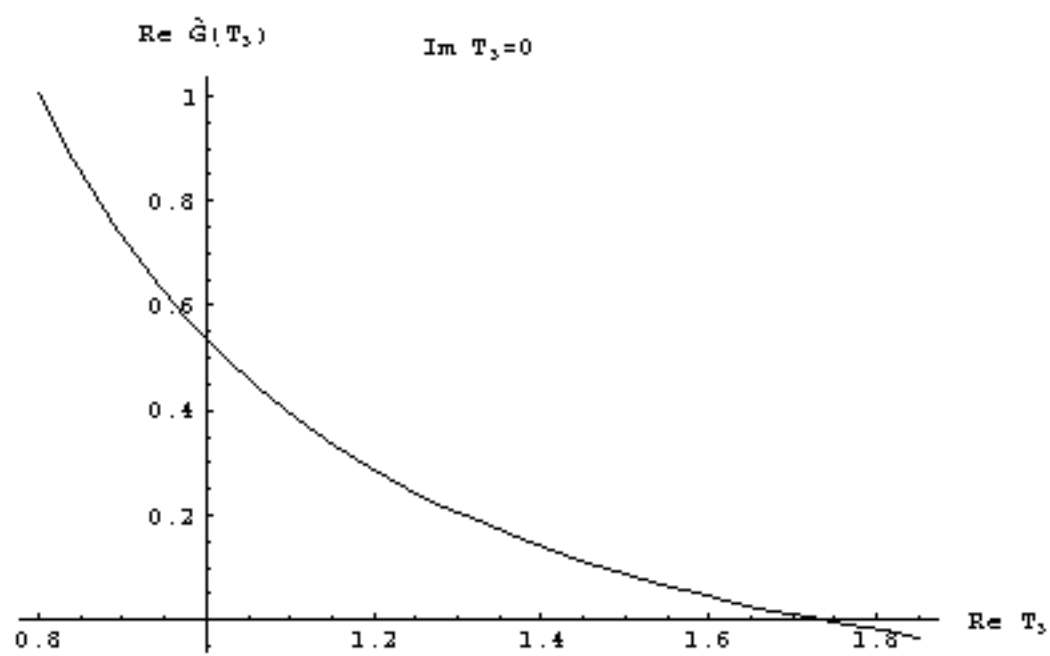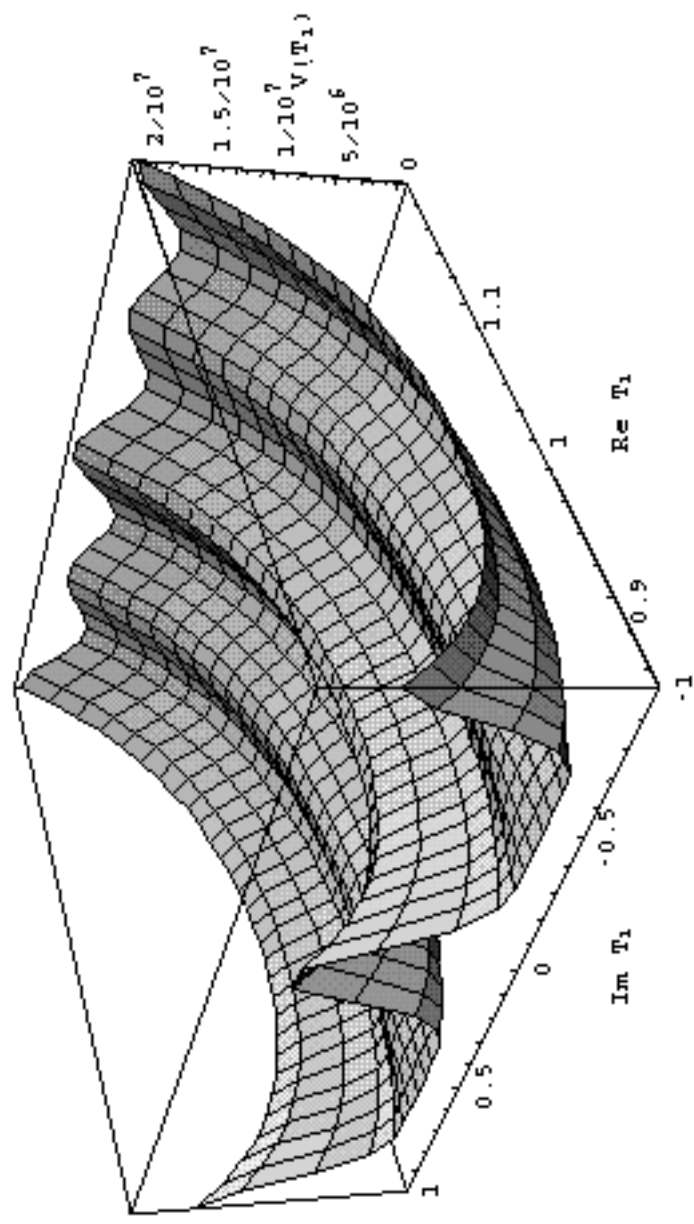
{ ∂_r, P=-1/4, $\partial_{r,r}$P=0.6, m1=n1=1, m3=n3=n4, m4=0, $\langle\zeta_{\varphi}(T_1)\rangle$=-10 }

Im $\hat{G}(T_3)$

Re $T_3 = \sqrt{3}/2$

0.02

0.01

Im T_3

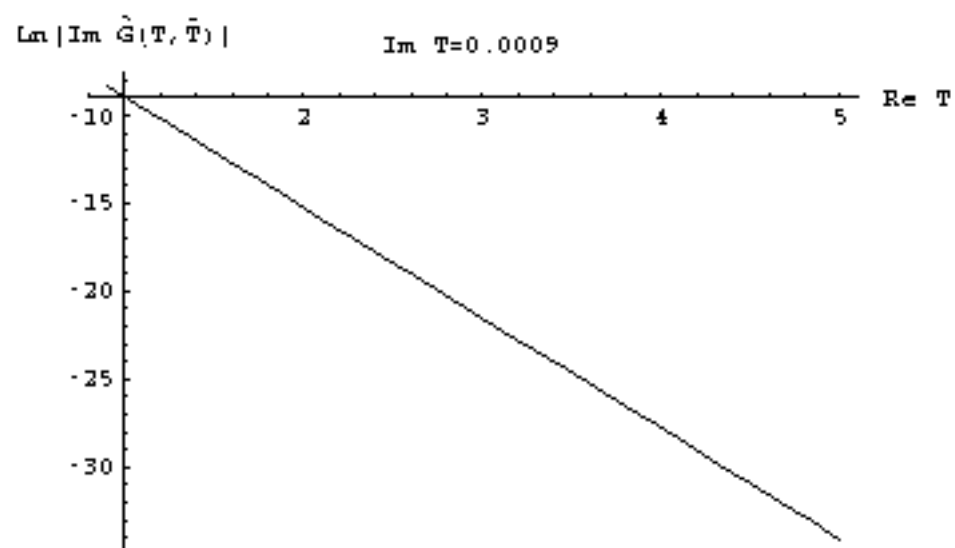1.425   1.45   1.475   1.5   1.525   1.55

-0.01

-0.02

-0.03

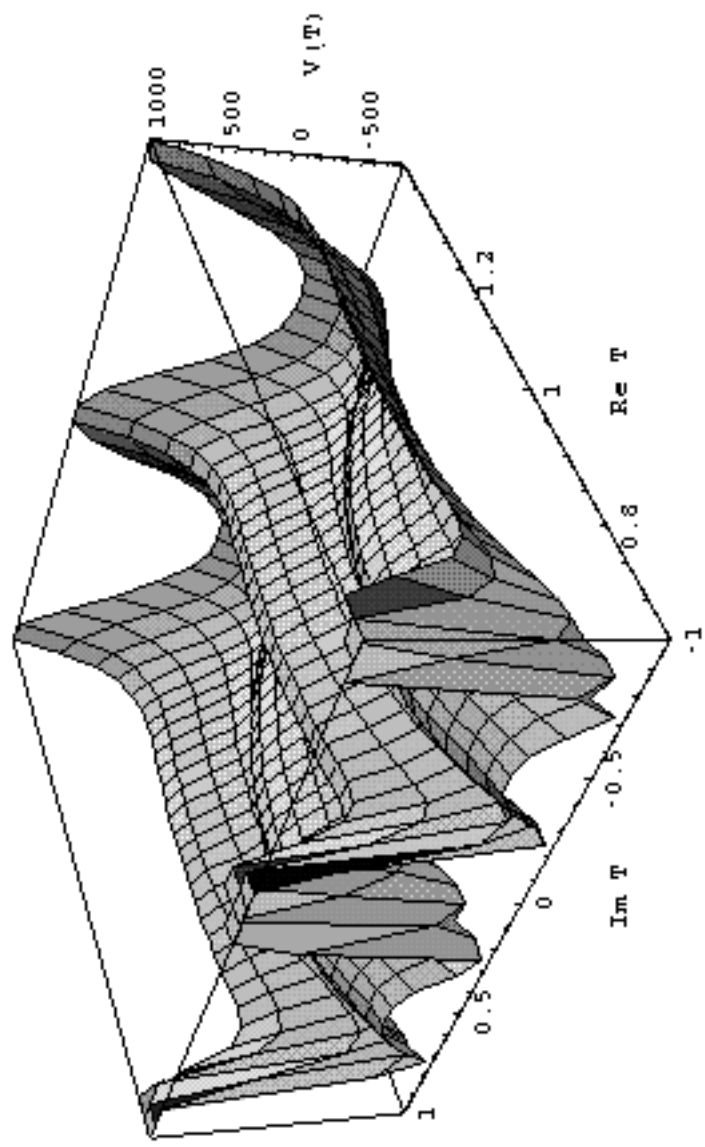$\{m(T_1)=n(T_1)=1, m(T_3)=n(T_3)=m(U_3)=n(U_3)=0,$ $\zeta_{vac}(T_1)=-10, \zeta=0.5\}$

$\{m(T_1)=n(T_1)=1, \partial, P=-2.45, \partial, \partial, R=-0.45, \qquad \zeta_{uw}(T_1)=-20, m(T_j)=n(T_j)=0, \quad \text{for } T_i \neq T_1\}$

{P(y) model,∂_y P=-1.4,$\partial_{y,y}$P=-0.1,m=1,n=3, ζ_{ss}=-30 }